

For the following questions, you may use the formulae for $\frac{d}{dx} \sinh x$ and/or $\frac{d}{dx} \cosh x$ without proving them. SCORE: _____ / 6 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] Without using the logarithmic formula for $\cosh^{-1} x$, prove the formula for $\frac{d}{dx} \cosh^{-1} x$.

$$y = \cosh^{-1} x$$

$$\cosh y = x$$

$$\textcircled{1} (\sinh y) \frac{dy}{dx} = 1$$

$$\textcircled{1} \frac{dy}{dx} = \frac{1}{\sinh y} = \frac{1}{\sqrt{x^2 - 1}} \textcircled{\frac{1}{2}}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\textcircled{\frac{1}{2}} x^2 - \sinh^2 y = 1$$

$$\sinh^2 y = x^2 - 1$$

$$\text{BONUS } \textcircled{1} \sinh y = \pm \sqrt{x^2 - 1}$$

GRAPH OF $\cosh x$
IS 1-1 IF DOMAIN RESTRICTED
TO $x \geq 0$

SO RANGE OF $\cosh^{-1} x$ IS $y \geq 0$
SO $\sinh y \geq 0$

[b] Prove the formula for $\frac{d}{dx} \coth x$.

$$\frac{d}{dx} \frac{\cosh x}{\sinh x}$$

$$= \frac{(\sinh x)(\sinh x) - (\cosh x)(\cosh x)}{\sinh^2 x}$$

$$\textcircled{1} \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} \textcircled{1}$$

$$= \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{-1}{\sinh^2 x} = -\operatorname{csch}^2 x \textcircled{\frac{1}{2}}$$

Find $\lim_{x \rightarrow -\infty} \coth x$. Do NOT use a graph. Give **BRIEF** algebraic or numerical reasoning.

SCORE: _____ / 3 PTS

$$= \lim_{x \rightarrow -\infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

OR

$$= \left[\lim_{x \rightarrow -\infty} \frac{1 + e^{-2x}}{1 - e^{-2x}} \right] \textcircled{1} \quad \frac{\infty}{-\infty}$$

$$\textcircled{1} = \left[\lim_{x \rightarrow -\infty} \frac{e^{2x} + 1}{e^{2x} - 1} \right]$$

$$= \left[\lim_{x \rightarrow -\infty} \frac{-2e^{-2x}}{2e^{-2x}} \right] \textcircled{1}$$

$$\textcircled{1} = \left[\frac{0+1}{0-1} \right]$$

$$= [-1] \textcircled{1}$$

$$\textcircled{1} = [-1]$$

☆ GRADE AGAINST ONE

VERSION ONLY

If $\tanh x = -\frac{1}{3}$, find $\sinh x$.

SCORE: _____ / 4 PTS

You may use any hyperbolic identities from your textbook without proving them.

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \frac{1}{9} = \operatorname{sech}^2 x$$

$$\textcircled{1} \quad \boxed{\operatorname{sech}^2 x = \frac{8}{9}}$$

$$\textcircled{1} \quad \boxed{\operatorname{sech} x = \frac{2\sqrt{2}}{3}} \quad \text{SINCE } \operatorname{sech} x > 0 \text{ FOR ALL } x$$

$$\textcircled{1} \quad \boxed{\cosh x = \frac{3}{2\sqrt{2}}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \rightarrow \sinh x = \tanh x \cosh x$$

$$= -\frac{1}{3} \cdot \frac{3}{2\sqrt{2}} = \boxed{\textcircled{1} -\frac{1}{2\sqrt{2}} \text{ OR } -\frac{\sqrt{2}}{4}}$$

EITHER
ANSWER OK

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

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NOTE: This is NOT a question about derivatives.

$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\left[\frac{e^y - e^{-y}}{2} = x \right] \textcircled{\frac{1}{2}}$$

$$\text{LET } z = e^y$$

$$\frac{z - \frac{1}{z}}{2} = x$$

$$z - \frac{1}{z} = 2x$$

$$\textcircled{1} z^2 - 1 = 2xz$$

$$\underline{z^2 - 2xz - 1 = 0}$$

$$z = \left[\frac{2x \pm \sqrt{4x^2 + 4}}{2} \right] \textcircled{1}$$

$$z = \frac{2x \pm 2\sqrt{x^2 + 1}}{2}$$

$$e^y = \left[x \pm \sqrt{x^2 + 1} \right] \textcircled{\frac{1}{2}} \quad x - \sqrt{x^2 + 1} < 0$$

$$\left[e^y = x + \sqrt{x^2 + 1} \right] \textcircled{\frac{1}{2}}$$

$$y = \left[\ln(x + \sqrt{x^2 + 1}) \right] \textcircled{\frac{1}{2}} = \sinh^{-1} x$$

OK IF YOU USED e^y DIRECTLY INSTEAD OF z

Find $\frac{d}{dx} x^3 \sinh^{-1}(x^2)$. Simplify your final answer.

SCORE: _____ / 3 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$3x^2 \sinh^{-1} x^2 + x^3 \cdot \frac{1}{\sqrt{1+(x^2)^2}} \cdot 2x$$

$$= \underbrace{3x^2 \sinh^{-1} x^2}_{\textcircled{1}} + \frac{\underbrace{2x^4}_{\textcircled{1}}}{\underbrace{\sqrt{1+x^4}}_{\textcircled{1}}}$$